Sets
Intersection, Union, Null Set
$A \oplus B$ (symmetric difference) $= A \cup B - A \cap B$
Addition principal - $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$
$\bar{A} = U - A$ (complement of A)
Subsets
Sets containing other sets
Set builder notation
Cardinality

Functions and integers
Everywhere defined – everything in A is used and each element in A goes to only 1 element in B
Onto – every element in B can be gotten to with the function ie. Range(f) = B
1-to-1 – each element in B can be gotten to by at most one element in A
Invertible – 1-to-1 and onto
A function is invertible if its inverse (f^{-1}) is also a function
$g \circ f - (g \circ f)(a) = g(f(a))$
$f \circ g - (f \circ g)(a) = f(g(a))$
Floor – round down
Ceiling – round up
LCM = $2^{\max(a,b)} * 3^{\max(a,b)} * 5^{\max(a,b)} * \ldots \{\text{all primes}\}^{\max(a,b)}$
GCD = $2^{\min(a,b)} * 3^{\min(a,b)} * 5^{\min(a,b)} * \ldots \{\text{all primes}\}^{\min(a,b)}$
Euclid's alogorithm – $d = sa + tb$
Base conversion – alternate using / and %

Sequences
Sequence – order matters
Finite – countable, has specific start and end points
Infinite – has no end point. The book calls it countable, but how do you count infinity?
Countable – can be arranged in a list, has a start
Uncountable – anything not countable, an example is all real numbers between 0 and 1
Recursive – element depends on previous values, may be infinite, but has a specific starting point
Explicit – element depends only upon itself, has a specific starting point
String – sequence of letters, set corresponding to a sequence
Regular Expressions – defining a set of strings

Counting and Probability
Multiplication principle
Permutation – order matters $P(n,r) = n!/(n-r)!$
Combination – order doesn't matter $C(n,r) = n!/(r!(n-r)!)$
Permutation with repeats – $P(n,r) = n^r$
Combination with repeats – C(n,r) = rewrite this one as a regular combination using C(n+r-1, r)
Event (E) – the desired outcome or combination of outcomes
Sample space (S) – all possible outcomes
Probability – $P = |E|/|S|$
Pigeonhole principle

Matrices – Boolean and Regular

Add, ∧, ∨ - only exact same sizes

Multiply – MxN * JxK is possible only if N=J, result is a MxK matrix

Transpose – flip around the diagonal (first row becomes first column, etc). Is symmetric if $A = A^T$

Identity matrix – binary matrix where the diagonal is all 1's, all other values are 0, is always square

Inverse – only computable for a 2x2 matrix (bigger can be done, but not in this class)

Propositions and logical operations

Truth tables for logical operators

Statement – true or false declaration (not opinion, question, command, changing value, etc.)

Graphs

Matix in-degree = number of arrows into node, number of 1's in the column

out-degree = number of arrows out of node, number of 1's in the row

Paths

Cycle – begin and end at the same vertex

Connectivity relation showing all paths of all lengths

R^n path of length n

Relations

reflexive – R is reflexive if aRa for all a in A

irreflexive – R is irreflexive if a~~R~~a for all a in A

symmetric – aRb and bRa

asymmetric – aRb and b~~R~~a (a□b or both 0, diagonal is 0)

antisymmetric – if aRb and bRa then a=b, else aRb and b~~R~~a, or both 0

transitive – if aRb and bRc then aRc

Digraph representations of relations

Matrix representations of relations

Graphs

reflexive – all nodes need a cycle of length 1

irreflexive – no node can have a cycle of length 1

symmetric – all edges go both ways, cycles of length 1 are ok

asymmetric – no cycle of length 1, all edges are single path

antisymmetric – all edges between vertices are single path, cycle of length 1 is ok

transitive – if there is a path of length 2 from a to c, passing through b, then there must also be a path of length 1 between a and c. If no path of length 2 exists, it is still transitive.

Growth of Function

Big-Theta and Big-Oh Notation

Trees

root – first or top vertex in the tree, has a height of 0
leaf – bottom vertex, has 0 children
n-tree – all vertices have at most n children
complete – all vertices except leaves have the same number of children
balanced – height of all leaves differ by at most 1
sub-tree – any vertex of a tree may be partitioned off (with all children etc) to become a new tree

Grammar

$G=(V, S, v_0, \rightarrow)$
G – grammar
V – everything, similar to the universe
S – set of terminal symbols
N – set of non-terminal symbols
$\rightarrow$ - the production

Machine

M = (S, I, F) machine or (S, I, F, s_0, T) Moore
machine

S – state set
I – input set
F – state transition function
T – terminal state set
s_0 – starting state