SIMULATING PEDESTRIAN ROUTE SELECTION WITH IMPERFECT KNOWLEDGE

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Abstract:

Heuristic evaluation of possible route choices allows pedestrians to make decisions in a timely and efficient manner. The heuristic function used to evaluate the route and the subsequent route selection has a large impact on the egress time of the pedestrian. We implement several common heuristic functions using the PLEASE simulation model and allow these heuristics to be combined using weighted factors. When the total distance of a route is unknown, using a greedy strategy of selecting the shortest-leg first route is shown to be a poor choice. When combined with other heuristic estimates however, including shortest-leg first costs can help to decrease egress times. We show that for a variety of building layouts using a heuristic function based upon width, distance, signage and congestion levels leads to better egress times.

1 INTRODUCTION

In everyday life, pedestrians frequently utilize heuristics to make potentially complicated decisions within an allotted time frame. These heuristics often allow a person to make a reasonable decision without requiring extensive amounts of time and energy. One example arises from the evacuation of a burning building. Selecting the best route can mean the difference between life and death, but frequently insufficient information is used in making this decision. Currently, detailed information about pedestrian egress from actual emergencies is not widely available, and thus it is too costly, too dangerous, and impractical to determine what heuristics pedestrians use to select an egress route when in emergency situations. Pedestrian simulation models have been designed to address this problem.

Current pedestrian simulation models generally make one of two assumptions about pedestrian knowledge. Either the model assumes pedestrians have perfect knowledge of the building layout and are thus able to select the best route, or the model assumes that pedestrians only know about the visible routes and must make a decision based upon some heuristic (Pan, 2006; Thompson and Marchant, 1995; Gwynne et al., 2001). Models which assume perfect knowledge are clearly unrealistic for many situations as not

all pedestrians will be familiar with a given building layout. Yet models relying upon a heuristic may also be unrealistic in the amount of information provided to pedestrians. Typically such heuristic models consider distance, congestion or social comparison for making the route selection decision. In this paper, we show that using distance as the sole means of comparing visible routes leads to poor egress times when the total route distances are unknown Therefore, if total route distances are assumed to be unknown to the pedestrians, then other route selection heuristics must be used to supplant the unknown information.

In this paper, we consider the effect of various heuristic functions on pedestrian egress time for a variety of different building layouts. Determining a comprehensive list of heuristic functions, which pedestrians might use, is outside of the scope of this paper. However, several common factors including distance, signage, corridor width, congestion/usage, and common consensus are used to produce a variety of realistic heuristic functions.

2 RELATED WORK

In (Ozel, 2001), Ozel raises several pertinent questions regarding the issue of stress management and the

decision process. He suggests that pedestrians utilize various coping mechanisms (heuristics) to make a decision in a time-pressured environment. In particular, the familiarity of routes and the negative connotations of emergency exits are shown to have a large impact on the route choice of pedestrians in an emergency.

Hoogendoorn and Bovy use distance and congestion heuristics for their route-choice and activity scheduling model (Hoogendoorn and Bovy, 2004). Gwynne et. al. indicate that pedestrians maintain social roles and norms even during emergency situations (Gwynne et al., 2006). This may lead to pedestrians choosing a different route based on the choices of their peers. Similarly, Fridman and Kaminka develop a pedestrian simulation model based upon social comparison theory (Fridman and Kaminka, 2007).

Golledge ranks the preference and prevalence of several different heuristic values pedestrians use in navigation and route selection in an outdoor environment such as a college campus through the use of questionares and observations (Golledge, 1995). He finds that pedestrians prefer routes which are direct, quick, and easy to navigate with some preference being given to routes which are more aesthetically pleasing. In emergency situations, one would expect the characteristics of directness and quickness to remain prominent while the importance of scenery is irrelevant. Golledge also found that the route selection criteria used differed for various route layouts and that a combination of multiple criteria may give better results.

3 Heuristics

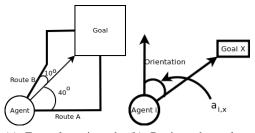
Our research evaluates a variety of heuristics. We describe several possible heuristics in the section.

Distance is a common heuristic used by pedestrians when making a route selection and is the primary heuristic in many search based strategies (Hoogendoorn and Bovy, 2004). Unless motivated by other factors, pedestrians will choose the route which has the shortest distance (Golledge, 1995). The distance heuristic is inapplicable in the absence of complete route distance knowledge because missing data causes the distance relation to be a partial order.

The shortest-leg first heuristic is a greedy distance heuristic. Rather than utilizing the total route distance, shortest-leg first selects the route with the shortest distance to the next decision point. Pedestrians only require distances to route goals which are within their field of view.

Another common heuristic used by pedestrians when selecting a route is the least angle heuristic

(Dalton, 2003; Hochmair and Frank, 2000). This heuristic selects the path which is closest in terms of angularity to a direct line between their current position and the final goal (Büchner et al., 2007). For example, in Fig. 1 the agent would select Route B when using the least angle heuristic because it is the route whose angle differs the least from the angle to the goal. Unfortunately, the end goal location(s) is not always known. In such situations, the least-angle heuristic cannot be applied.



(a) Example using the (b) Depicts the values least-angle heuristic used to calculate the angle heuristic

Figure 1: Least-Angle heuristic examples

We can modify the least angle heuristic to accommodate unknown goals by selecting the closest path in terms of angularity to the current walking orientation. This new heuristic is actually a greedy version of the fewest turns heuristic as it tries to minimize the number of direction changes an agent makes while exiting from the building.

Most buildings include navigational signs to assist pedestrians in locating their desired location (Kray et al., 2005). These signs might include exit signs, emergency exits signs, room signs, or navigational maps. Pedestrians then use these signs to navigate effectively through the building. One might think that during an emergency evacuation pedestrians would look for exit signs or emergency exit signs, but Ozel found that emergency exit signs often have negative connotation and are avoided even in emergency situations (Ozel, 2001). Exit signs, on the other hand, are commonly used by pedestrians in navigational planning when in an unfamiliar building. Room signs can also be used in egress route planning as they indicate that an egress route through a particular door is unlikely because individual rooms rarely contain egress routes.

Main corridors tend to be preferred by pedestrians especially when navigating an unfamiliar building (Hillier, 1996). Like the relative width of various categories of roads, main corridors are wider than auxiliary hallways so the width of the corridor or doorway can frequently be an indication of a main route of travel leading to an exit. The number of pedestrians currently gathered around a location may be either a positive or a negative indicator of a desirable exit route (Gwynne et al., 2006; Pan, 2006; Koh et al., 2008). It may indicate the route is a good choice because others are using it. However, it may also indicate that another route should be considered to avoid the congestion. Intuition suggests that if pedestrians are unsure about the situation, they will follow others. If they are more confident and know multiple routes, they will seek alternatives to avoid congestion.

Unless the pedestrians are traveling in a group, they will not know the exact route of nearby pedestrians. However, by observing the velocity and past movement, an agent may predict the immediate destination of a neighboring pedestrian. Seeing agents who are traveling in the same direction may bolster the confidence level of an agent in choosing a particular route (Ozel, 2001). Traveling in the same direction as others is also easier as one does not have to fight against the general direction flow, improving the overall pedestrian flow.

Pedestrians will typically choose routes with which they are familiar especially when under time constraints (Ozel, 2001). Familiarity allows the pedestrian to feel more comfortable and confident in the route selected and allows the pedestrian to concentrate on other cognitive tasks. However, selecting a new route may or may not lead to a better solution. When the current known route cost is within an acceptable limit, pedestrians are unlikely to change. The cost of a route may be based upon any number of factors such as distance, congestion, time, and so on. Readers may refer to (Feuz, 2011) for details of including distance and congestion costs as learned by past experience.

4 Simulation Environment

This study is performed using the Pedestrian Leadership and Egress Assistance Simulation Environment (PLEASE). PLEASE is built upon the multi-agent modeling paradigm where each pedestrian is represented as an individually rational agent capable of perceiving the environment and reacting to it. In PLEASE, pedestrian agents can perceive obstacles, hazards, routes, and other agents. The agents use a two tier navigational module to control their movement within the simulation environment. The high-level tier evaluates available routes and selects a destination goal. The low-level tier, based on the social force model (Helbing and Johansson, 2009), performs basic navigation and collision avoidance.

PLEASE implements several of the heuristics outlined in Section 3. Here we describe the implementation details used for the heuristics of interest. Many simulation models assume that total route distance is known and/or the end goal location is known for at least some subset of the available routes. However, in this paper, we are interested in the case when no additional information, beyond what is immediately visible to the pedestrian, is provided. For this reason, the distance, angle and learned-costs heuristics are not considered.

Each heuristic can potentially be combined with any other heuristic. To facilitate the integration of multiple heuristics, all heuristic values are normalized so that the unweighted cost falls between 0 and 1.

The leg cost of route x, L(x), is given by Formula 1 where w_l is the weight applied to the shortest-leg heuristic, and $d_{i,x}$ is the distance from agent i to route goal x. The distance is normalized using the maximum distance between two points on the simulation map. Agents in the simulation are able to accurately estimate the distance to visible points within the simulation model. The distance to locations which are occluded by walls or other obstacles cannot be estimated without prior knowledge.

$$L(x) = \frac{w_l * d_{i,x}}{(maxDistance)}$$
 (1)

The turn cost of route x, T(x), is given by Formula 2, where w_t is the weight applied to the fewest turns heuristic, and $a_{i,x}$ is the angle in radians between the orientation of agent i and the direction to route goal x from agent i. π acts as the normalization factor since no angle will be greater than π . See Figure 1 for an example.

$$T(x) = w_t * a_{i,x}/\pi \tag{2}$$

The signage cost of route x, S(x), is given by Formula 3, where w_s is the weight applied to the signage heuristic, and getExitWeight(x) is the cost associated with the given signage value. getExitWeight takes the signage of a route and looks up the user-specified cost of that sign. A user may specify arbitrary sign costs, but by default, PLEASE uses the costs shown in table 1. To be consistent with the other heuristics, cost should be specified as a value between 0 and 1.

$$S(x) = w_s * getExitWeight(x)$$
 (3)

The simple signage cost of route x, SS(x), is given by Formula 4, where w_{ss} is the weight applied to the simple signage heuristic, and getSimpleSignage(x) is the cost associated with the simple signage of route x. A route has a simple signage cost of 0 if the doorway of the route is a direct exit and is marked with an exit

Table 1: Default parameters used in PLEASE

Sign Type	Cost	Width	Cost
Emergency Exit	0.25	Small	1.0
Exit	0.0	Medium	0.5
Room	1.0	Large	0.0
None	0.5		

sign. All other routes have a simple signage cost of 1. This causes a pedestrian to ignore all building signage except for exit signs over direct exits. The simple signage heuristic allows us to compare the effect that different amounts of building signage and different levels of attention to the building signage have upon the egress times of pedestrians.

$$SS(x) = w_{ss} * getSimpleSignage(x)$$
 (4)

The width cost of route x, W(x), is given by Formula 5, where w_w is the weight applied to the corridor width heuristic, and getCorridorWeight(x) is the cost assigned to the given corridor width. The getCorridorWeight is a lookup table which takes the width of a route and returns the user-specified cost for that width. A user may enter any arbitrary width cost, but by default, PLEASE uses the costs shown in table 1. To be consistent with the other heuristics, cost should be specified as a value between 0 and 1. Although corridor width is a real number and can have an infinite number of values, the getCorridorWeight discretizes width into three categories, small, medium and large. This is done to eliminate meaningless differences between corridors. Two corridors which differ only slightly in width are equally likely to indicate a main route and should be treated equally. The cutoff values for these categories can be set by the user so that the values are appropriate for the current building layout. By default, PLEASE uses 1.5 m and 2.5 m as the cutoff values. Agents in the simulation measure width at the entry point of the corridor.

$$W(x) = w_w * getCorridorWeight(x)$$
 (5)

The congestion cost of route x, Cong(x), is given by Formula 6, where w_{cg} is the weight applied to the congestion heuristic, sp_i is the desired speed (a normally distributed parameter value unique to each pedestrian) of pedestrian i, sp_j is the speed of pedestrian j, s_1 is 1 if $sp_j < sp_i$ or 0 otherwise, $n_{p,x}$ is the number of pedestrians along route x, and n_p is the total number of pedestrians. This formula assigns cost based upon the desired speed of the pedestrian and the current speed of pedestrians along the selected route. For each pedestrian along the selected route, if their speed is slower than the desired speed, then a cost is incurred relative to the speed difference. The cost is raised to the square so that smaller speed differences count less than larger differences. Finally, the result is normalized by the worst case cost (i.e. if every pedestrian in the simulation was along the selected route and was not moving).

$$Cong(x) = w_{cg} * \frac{\sum_{j=0}^{n_{p,x}} ((sp_i - sp_j) * s_1)^2}{sp_i * n_p}$$
 (6)

The consensus cost of route x, C(x), is given by Formula 7, where w_c is the weight applied to the consensus heuristic, $a_{j,x}$ is the angle between pedestrian j's orientation and the direction to route goal x from agent j (see Figure 1), and $n_{p,i}$ is the number of pedestrians surrounding pedestrian i.

$$C(x) = w_c * \frac{\sum_{j=0}^{n_{p,i}} (a_{j,x}/\pi)}{n_{p,i}}$$
 (7)

The previously visited cost of x, V(x), is given by Formula 8, where w_v is the weight applied to the visited heuristic, and visitedCount(x) is the number of remembered times route x has been visited by the pedestrian in this simulation run. Each pedestrian is capable of remembering a specified limit of number of routes for a specified amount of time to reflect the finite memory of pedestrians. These limits may be set to any arbitrary value by the user, but by default, PLEASE has a limit of 10 routes for 1000 seconds.

$$V(x) = w_v * visitedCount(x)/maxMemory$$
 (8)

5 Experimental Design

To test the effectiveness of the various heuristics described above, we use a combination of actual building layouts and building layouts constructed for the purpose of these experiments. In the buildings shown in Figure 2, blue lines represent walls of the buildings, dashed green lines represent exits signs, solid red lines represent emergency exit signs, and wavy gold lines represent door signs. The USU Business building (see Figure 2(a)) is an approximation of the ground floor of the actual building found on the Utah State University Campus. Likewise, the CSULB FM building (see Figure 2(b)) and the CSULB UP building (see Figure 2(b)) are approximations of the actual buildings found on the California State University, Long Beach campus.

The USU Business building and the CSULB UP building are used because the floor plans follow expected conventions: rooms do not function as hall-

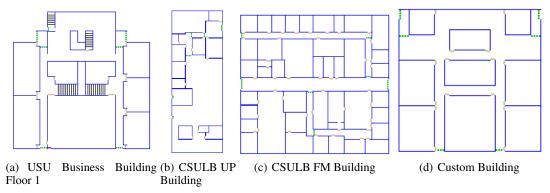


Figure 2: Building layouts used in the heuristic evaluation experiments.

ways, main areas have wide corridors, and other accepted conventions are followed. The CSULB FM building is used because of the lack of convention with room placement. Rooms are found within rooms and several rooms can even function as hallways. The custom building is designed to represent a standard building. The corridor widths are not an exact indication of main routes and areas, but are still closely correlated to main routes. Rooms do not function as hallways. The number of rooms and routes are not so many as to be completely unmanageable by pedestrians either.

For each building, we conduct a variety of tests. We measure the total egress time of 100 pedestrians, randomly distributed throughout the rooms, averaged over 20 simulations using a single active heuristic. In all the tests, the previously visited heuristic is always active and is given a weight of five. This represents the agents' unwillingness to backtrack along a route previously traveled.

Based upon the performance of the individual heuristics, we then combine multiple heuristics, weighting each heuristic by its relative performance in relation to the total egress time achieved by the heuristics. These weights are then adjusted to further improve the performance of the combined heuristics. Determining the exact weight specification to optimize performance when multiple heuristics are used is outside of the scope of this paper.

6 Results and Analysis

In this section, we discuss the results of the experiment in four parts. First, we look at the effect of building layouts on egress times. Second, we consider in detail the result of applying only one heuristic at a time. Third, we look at the results of applying multiple heuristics simultaneously. Fourth, we rank each heuristic using a relative performance ratio.

6.1 Building Layouts

The USU Business building and the CSULB-UP building have similar results for several of the heuristics (see Figure 3). This makes sense because both buildings have similar design characteristics. The main differences in heuristic performance occurred with the shortest-leg heuristic. The USU Business building layout happens to be conducive to using the shortest-leg heuristic. The outside rooms can actually be used as hallways which lead directly to the exits, and this is exactly what happens when the shortest-leg heuristic is used. The ground floor of the USU Business building layout appears similar to the custom building layout, yet the slight differences in spacing and room layout create large differences in egress times for the two buildings.

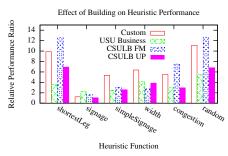


Figure 3: Effect of building layout on heuristic effectiveness. Performance ratio is calculated as the average time taken for 90% of the pedestrians to evacuate using the given heuristic divided by the average time taken for 90% of the pedestrians to evacuate using perfect knowledge.

The CSULB FM building and the custom building both have similar results in egress times for the shortest-leg, signage, congestion and random heuristics. Although the actual layouts of the building are quite different, the underlying patterns are similar: width correlates with egress routes, long hallways have many adjoining rooms, and exits are distributed

in a uniform manner. The main difference between the results for each building is the performance of the width heuristic. In the CSULB FM building, corridor width corresponds closely with egress routes, while in the custom building the correlation is weaker. The simple signage heuristic also yields different results in these two buildings. In the custom building, the top left and top right exits are not visible throughout most of the building and are thus highly underutilized. Meanwhile, the exits in the CSULB FM building have a much higher level of visibility throughout the building and are used more effectively.

6.2 One Heuristic

When only a single heuristic is used, the results vary greatly between heuristics (see Figure 4). The signage heuristic performs well in all types of building layouts tested. This suggests that if pedestrians choose an egress route based upon well-designed signage, pedestrians can efficiently egress from a building even when completely unfamiliar with the layout of the building. Using simple signage, the egress times are still as good as or better than any other heuristic in the building layouts considered. This highlights the importance of even minimal building signage in assisting in pedestrian egress.

The shortest-leg heuristic leads to slow egress times in almost every building layout considered. In many instances, the shortest-leg heuristic does not even outperform a random choice policy. As discussed in Section 3, when the end goal is not known, choosing the route which is closest to the pedestrian becomes the shortest-leg first heuristic. This greedy route selection heuristic provides no guarantee that the route chosen will even lead to a direct exit. Additionally, when distance is the sole means of evaluating a route, congestion is a common occurrence. Pedestrians who are closest to a given doorway will select that doorway regardless of what side they are on or which direction other pedestrians are moving. Thus, the pedestrians on opposite sides of the doorway will converge at the doorway causing a bottleneck, and pedestrian flow rates through the doorway will be greatly inhibited. Interestingly, the shortestleg heuristic performs remarkably well in the USU Business Building (see Figure 4). This building is configured ideally so that greedily selecting the closest visible route actually leads pedestrians to an exit in a fairly efficient manner. One reason that this is the case is the double doors on most of the rooms. This allows the pedestrian to explore routes without having to backtrack, which is discouraged by the algorithm. Additionally, the end rooms have doorways adjacent to exits, which facilitates egress in this situation.

Similar to the shortest-leg heuristic, the fewest turns heuristic also leads to poor performance. The results are not shown here. The intuition behind the fewest turns heuristic is to select a route that is as direct as possible. However, considering only the next route goal is too short-sighted and leads to routes which are drastically less direct than they could be. Without prior knowledge about the building layout, though, this short-sightedness cannot be overcome.

For the building layouts considered in this paper, the width heuristic leads to average egress times when compared to the other heuristics. For the CSULB-FM building, choosing the widest route leads to finding an exit sooner than selecting a route by any other heuristic except for signage. In the remaining buildings, the width heuristic performs worse than the congestion heuristic, but still significantly outperforms a random policy.

The congestion heuristic does not necessarily provide an indication of which route leads to an exit, especially when none of the pedestrians have any knowledge regarding the building layout. However, avoiding congestion still improves the overall egress time by helping prevent bottlenecks and increasing the overall smoothness of pedestrian flow. This allows more routes to be explored in less time, which leads to better egress times. Although (due to space limitations) the results are not shown here, the consensus heuristic also relieves congestions at bottlenecks and improves pedestrian flow so that routes can be explored in a more efficient manner. To be most effective, the congestion and consensus heuristics should be combined with another heuristic such as width or signage which provide an indication of an egress route.

6.3 Multiple Heuristics

After considering each heuristic individually, we then combine several heuristics to further improve performance (see Figure 5). Although many different combinations could be tried, in this paper, we consider combining the simple signage and width heuristics (SS-W) and the shortest-leg, simple signage, width, and congestion (SL-SS-W-C) heuristics. When width is the sole heuristic applied, the egress times are too slow to be reliable in an emergency. The simple signage heuristic is also slower than desired but is still the best alternative to the signage heuristic (which may not be realistic for many buildings) when only a single heuristic is applied. The goal of combining width and simple signage is to take advantage of the signage when available and to fall back on the width

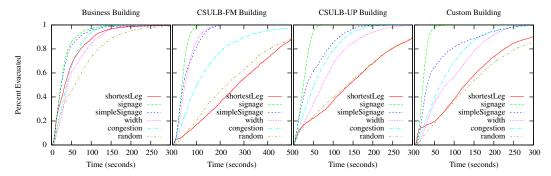


Figure 4: Agent egress times using multiple heuristics in the Simple building

heuristic when the signage is not available. We then include the other heuristics, namely shortest-leg and congestion, to further improve the egress times. For comparison purposes, the egress times of the signage, simple signage and width heuristics are included in the charts.

As can be seen in Figure 5, combining the simple signage and width heuristic (SS-W) did indeed improve performance in most buildings in comparision to either heuristic alone. The custom building layout is the one exception. In this building, falling back on width proved to be detrimental to the overall egress time as the widest areas did not have direct exits. However, when several heuristics (shortestleg, simple signage, width and congestion, denoted as SL-SS-W-C) are combined, performance is improved in every single building layout when compared to the performance of the heuristics separately. In most cases, the egress times matched or beat selecting routes based upon perfect signage. This indicates, that for a wide variety of buildings, the same heuristic functions can be applied to successfully egress from the building in a reasonably efficient manner using only the information which is directly perceivable by the pedestrian.

Although the shortest-leg heuristic did not perform well by itself, when combined with other heuristics, it leads to improved performance (results omitted due to space limitations). This is indicative of the value distance can play in route selection and justifies its use by actual pedestrians. However, it is important to note the disastrous impact which relying only upon distance can have upon the total egress time of individual pedestrians when no additional information is utilized.

6.4 Heuristic Rankings

The heuristic functions are ranked according to the relative performance ratio (RPR) of each heuristic in

the above mentioned building layouts. For each building, the average time (t) it takes for 90% of the pedestrians to evacuate when each pedestrian has perfect knowledge of route distances and congestion levels is recorded. The average time (h) it takes for 90% of the pedestrians to evacuate when each pedestrian is using the heuristic function of interest is also recorded. The RPR of each heuristic function is then computed as h/t. Table 2 displays the average relative performance ratio for each heuristic.

Table 2: Ranking of heuristic functions by relative performance ratio (RPR). A lower RPR signifies better egress times.

Heuristic	RPR	Heuristic	RPR
signage	1.44	congestion	4.30
S-SS-W-C	1.59	consensus	4.65
SS-W	2.71	random	7.60
simpleSignage	2.89	shortestLeg	8.19
width	3.62	angle	9.41

7 Conclusion

Using heuristic estimations in selecting an egress route is a natural and common process performed by pedestrians on a daily basis, yet most simulation models do not adequately address this fact. This paper highlights the importance of including heuristic costs in pedestrian simulators, especially those designed to model egress in an emergency situation. As Ozel indicates in (Ozel, 2001), when in stressful situations and under time constraints, pedestrians will react by filtering and bolstering information (i.e. relying more upon heuristic estimations). This leads to decisions which are sub-optimal and, as shown, can have a significant impact on the total egress time of the simulation. In emergency simulation models, it is not sufficient to assume pedestrians will make the best or even good choices, the simulation model needs to consider the

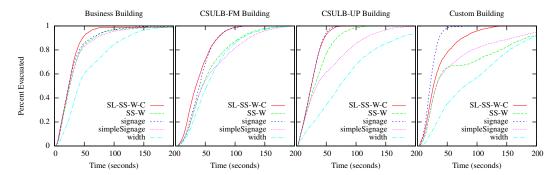


Figure 5: Agent egress times using multiple heuristics in the Simple building

possibility that pedestrians are forced to make splitsecond decisions with little information.

When the total distance is unknown, using a greedy strategy of selecting the closest route available is seen to produce poor results in many circumstances. If additional factors are included in the decision, then the distance heuristic can help improve egress times, even when the total distance is not known. Using four main heuristics (the shortest-leg heuristic, the simple signage heuristic, the width heuristic and the congestion heuristic, each appropriately weighted) is shown to produce good egress times even when no information about the building layout is known before the simulation begins. If only a single heuristic is used, the signage heuristic gives the best results even when the amount of building signage is minimal.

REFERENCES

- Büchner, S., Hölscher, C., and Strube, G. (2007). Path choice heuristics for navigation related to mental representations of a building. In *Proceedings of the European Cognitive Science Conference*, pages 504–509. Taylor & Francis.
- Dalton, R. (2003). The secret is to follow your nose. *Environment and Behavior*, 35(1):107.
- Feuz, K. (2011). Pedestrian leadership and egress assistance simulation environment. Master's thesis, Utah State University.
- Fridman, N. and Kaminka, G. (2007). Towards a cognitive model of crowd behavior based on social comparison theory. In *Proceedings of the National Conference on Artificial Intelligence*, volume 22, page 731. Menlo Park, CA; Cambridge, MA; London; AAAI Press; MIT Press; 1999.
- Golledge, R. (1995). Path selection and route preference in human navigation: A progress report. *Spatial Information Theory A Theoretical Basis for GIS*, pages 207–222.

- Gwynne, S., Galea, E., and Lawrence, P. (2006). The introduction of social adaptation within evacuation modelling. *Fire and materials*, 30(4):285–309.
- Gwynne, S., Galea, E. R., Lawrence, P. J., and Filippidis, L. (2001). Modelling occupant interaction with fire conditions using the buildingexodus evacuation model. *Fire Safety Journal*, 36(4):327–357.
- Helbing, D. and Johansson, A. (2009). Pedestrian, crowd and evacuation dynamics. In *Encyclopedia of Complexity and Systems Science*, pages 6476–6495. Springer.
- Hillier, B. (1996). *Space is the Machine*. Cambridge University Press, Cambridge.
- Hochmair, H. and Frank, A. (2000). Influence of estimation errors on wayfinding-decisions in unknown street networks—analyzing the least-angle strategy. *Spatial Cognition and Computation*, 2(4):283–313.
- Hoogendoorn, S. and Bovy, P. (2004). Pedestrian routechoice and activity scheduling theory and models. *Transportation Research Part B: Methodological*, 38(2):169–190.
- Koh, W. L., Lin, L., and Zhou, S. (2008). Modelling and simulation of pedestrian behaviours. In PADS '08: Proceedings of the 22nd Workshop on Principles of Advanced and Distributed Simulation, pages 43–50, Washington, DC, USA. IEEE Computer Society.
- Kray, C., Kortuem, G., and Krüger, A. (2005). Adaptive navigation support with public displays. In *Proceedings of the 10th international conference on Intelligent user interfaces*, IUI '05, pages 326–328, New York, NY, USA. ACM.
- Ozel, F. (2001). Time pressure and stress as a factor during emergency egress. *Safety Science*, 38(2):95–107.
- Pan, X. (2006). Computational Modeling of Human and Social Behaviors for Emergency Egress Analysis. PhD thesis, Stanford University, Stanford, California.
- Thompson, P. and Marchant, E. (1995). Testing and application of the computer model simulex. *Fire Safety Journal*, 24(2):149–166.